Physics II
ISI B.Math
Back Paper Exam

Total Marks: 100
Answer any 5 questions.

1. The heat capacity of nonmetallic solids at sufficiently low temperatures is proportional to T^3, as $C = aT^3$. Assume it were possible to cool a piece of such a solid to $T = 0$ by means of a reversible refrigerator that uses the solid specimen as its low-temperature (variable!) reservoir, and for which the high temperature reservoir has a fixed temperature T_h equal to the initial temperature T_i of the solid. Find an expression for the electrical energy required.

2. Consider a spring which follows Hooke's Law: namely the elongation x is proportional to the tension X when it is pulled at constant temperature. The proportionality constant (spring constant) k is temperature- dependent. Determine the free energy F, the internal energy U, and the entropy S as a function of x. Neglect the thermal expansion.

3. (a) Show that the internal energy of an ideal gas is a function of temperature only.
(b) Find the change in entropy when an ideal gas at temperature T undergoes an adiabatic free expansion into vacuum from volume V_1 to volume V_2.
(c) Show that the following relation exists between the adiabatic compressibility $\kappa_{ad} = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{ad}$ and the isothermal compressibility $\kappa_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$

$$\kappa_{ad} = \frac{c_v}{c_p}\kappa_T$$

where v is the specific volume p is the pressure and c_p and c_v are the (constant) specific heats at constant pressure and volume.

4. Consider a system of N weakly interacting particles each of which has two energy levels, one with energy 0 and the other with energy ϵ. The system is in contact with a heat reeservoir at temperature T. Find the
(a)The average energy $\bar{E}$ of the system.
(b)The heat capacity C_V at constant volume.

(c) Show that the fluctuations in energy characterized by $\frac{\Delta E}{\bar{E}}$ is small when N is large.
($\Delta E = \sqrt{< (E - \bar{E})^2 >}$).

5. For electromagnetic radiation in thermal equilibrium at temperature T in a metallic cavity the energy of a mode of angular frequency ω is given by $\epsilon_n = n\hbar\omega$ where n is the number of photons in the mode . $(n = 0, 1, 2, 3, \cdots)$.
(a) Find the average number of photons in a single mode of frequency ω.
(b) Find the average energy in the above mode. What is the classical limit of the average energy?
(c) Given that the spectral density of this radiation, i.e, the energy density per unit angular frequency range is given by

$$u_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp\frac{\hbar\omega}{kT} - 1}$$

show that the entropy density of photons at temperature T is proportional to T^3. [You may need to use $\int_0^\infty dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15}$]

6. (a) Using the canonical distribution, assuming an isothermal atmosphere at temperature T show that the probability $P(z)$ of finding a molecule of mass m at a height z of the atmosphere is given by

$$P(z) = P(0)e^{\frac{-mgz}{kT}}$$

(b) Consider an isolated system of N particles with total energy E. Each individual particle can have energy $-\epsilon_0$ or ϵ_0. Find temperature of the system.